Material and Momentum Transport in Axisymmetric Turbulent Jets of Water

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The spread of momentum and heat in turbulent axisymmetric jets of incompressible fluids has been examined by numerous investigators since Tollmien, using Prandtl's mixing length theory, first approximated the solution to the transport problem in 1926. The reasons for the interest in this flow system are at least twofold: the jet is a technologically important flow, appearing in many large-scale chemical and physical operations, and the jet is a relatively simple shear flow regime for which, ostensibly at least, dynamic similarity is achieved for the various transport processes, that is, of heat, material, and momentum. It is the latter point with which most of the work has been concerned.

Much of the experimental work reported in the literature concerns jets of air and the relative rates at which momentum and heat spread out downstream of the orifice. Less has been done with material transport, primarily because local concentration measurements are always difficult to make. Corrsin and Uberoi (*1*) measured momentum and heat transport rates in turbulent jets of air and found the turbulent Prandtl number (the ratio of turbulent momentum transport to turbulent heat transport) to be 0.74, the same as the Prandtl number for transport by molecular motion. Hinze and van der Hegge Zijnen (*2*), Ruden (*3*), and Reichardt (*4*) report similar measurements on jets of air, and these agree well with a turbulent Prandtl number of 0.74. Hinze and van der Hegge Zijnen also measured the rate of spread of material (1% city gas in air) in their system and found that it spreads at the same rate as heat. They obtained a turbulent Schmidt number (the ratio of turbulent momentum transport to turbulent material transport) of 0.74. Keagy and Weller (*5*) obtained a value of 0.72 for a nitrogen jet issuing into still air and Forstall and Shapiro (*6*) a value of 0.71 for a jet of helium-in-air issuing into a moving stream of air.

The evidence, then, is that material and heat are transported in turbulent air jets at about the same rate, a rate which is appreciably greater than that for momentum. It is generally accepted that the explanation for this difference lies in the detailed structure of the turbulence which until only relatively recently was unknown. For a detailed discussion of the precise role played by the turbulence in transport processes, see the paper by Corrsin and Kistler (*7*) or the book by Townsend (*8*).

Although different results are not expected, still there have been few measurements reported for turbulent material and momentum transport rates in liquids. The chief paper here is probably that of Forstall and Gaylord (*9*) who used a jet of fresh water issuing into a 1% solution of sodium chloride. The general behavior of this jet was the same as that observed for jets of air, that is, the jet spread linearly with distance downstream, etc., but they obtained turbulent Schmidt numbers ranging between 0.75 and 0.85. Their best value appears to be about 0.8, a value significantly larger than the 0.71 to 0.74 obtained for gas jets. Forstall and Gaylord offer no comment on this other than that it may be due to some experimental error. Owing to the difficulty of making local concentration measurements, this may very probably be the case. Measurements of turbulent heat transport rates in this medium would be of value here, but none appear to be reported in the literature.

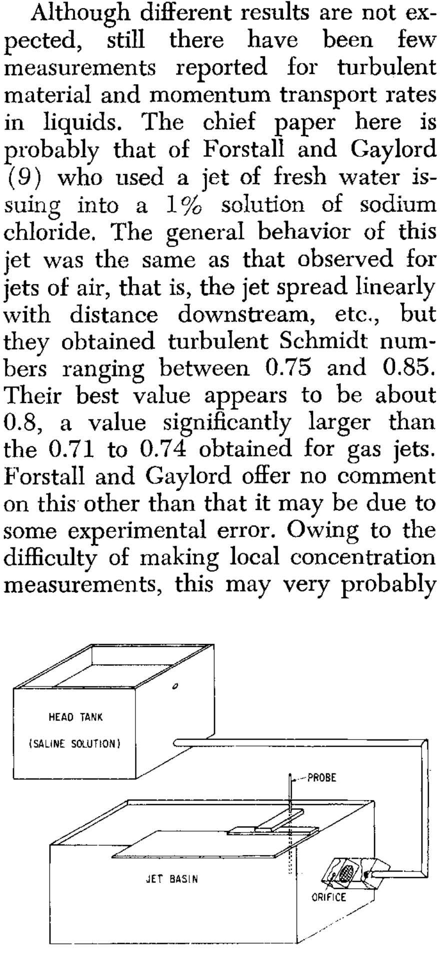

Fig. 1. Schematic diagram of the jet system.

For several reasons, there is justification for adding the present measurements to the literature. Much of the available data are taken from heated air jets, jets which are not entirely free from buoyancy effects, particularly where temperature differences are large (as they must be in order to make the measurements.) This problem is easily avoided by using liquid jets with concomitant material transport. There is the advantage of the rate of molecular material transport in liquids. This is slower by orders of magnitude than molecular heat and material transport in gases. There is, therefore, greater assurance that only the turbulent transport phenomena are being measured.

In general, the object of the present effort has been to examine the relative rates at which momentum and material are transported in a turbulently flowing jet of water when, ostensibly at least, the conditions are such that complete analogy exists; to determine whether or not the reasonable assumption that the relative rates are independent of the fluid is a valid one, and to test an instrument which is designed to provide detailed information on the structure of the turbulently fluctuating concentration field in the jet.

In this report only the structure of the mean flow and the mean concentration field is considered. A later paper will consider the structure of the randomly fluctuating concentration field.

EQUIPMENT FOR MEASUREMENTS

Jet System

A schematic of the ⅜ in. submerged jet is shown in Figure 1. The jet itself was a dilute solution of sodium chloride driven by gravity from a large head tank into a quiescent larger volume of only slightly less salt concentration, the latter being contained in a receiving basin 4 ft. long × 2 ft. wide × 2 ft. deep. A constant level was maintained in this basin by an overflow system (not shown). The jet issued horizontally through a 1/16-in. thick sharp-edged orifice located about 1 ft. below the surface. Because of the large volume of ambient fluid, and because of the small difference in salinity between the jet and the ambient fluid, the salinity of the ambient fluid rose but slightly in the course of a measurement. For these measurements, the velocity of efflux from the orifice was 12 ft./sec., giving a Reynolds number, based on the diameter of the orifice, of about 30,000. Mean velocity and concentration profiles near the orifice were measured and found to be rectangular within the accuracy of the measurements.

Measuring Equipment

Mean velocities were determined from measurements made of the total head using a 24-gauge hypodermic tube. Measurements were made at intervals of 1/32 and 1/16 in. (depending on the distance from the orifice) at various stations in the jet.

Concentration measurements were made using a conductivity cell and a sensitive detector circuit shown schematically in Figure 2.

Signal detection, in principle at least, is the same as that first used by Prausnitz (*10*) and later by Cairns (*11*) and Manning (*12*). This detector is, however, linear over a wider range of concentrations, and its frequency range is from d.c. to about 5 kcycles/sec. The precise details and characteristics of this circuit will be presented at a later date. The measurements were taken continuously by traversing slowly (as slowly as 0.5 in./min.) across the jet.

The output from the detector (~ fluctuating concentration) was fed to an impedance matching amplifier, and the mean value of this signal recorded. The output of the detector was also read directly with a vacuum tube voltmeter or was displayed on an oscilloscope for visual inspection.

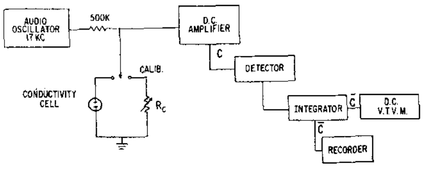

Fig. 2. Schematic diagram of the electronic circuitry.

A conductivity cell was used identical to that used in reference *12*, except that in this work the receiving basin serves as the distant electrode. The series capacitance of the cell was measured in situ and was found to be less than 1 $\mu\mu$ *F*. Impedance losses were therefore negligible for frequencies well beyond those encountered in this work.

RESULTS

Except for the axial traverses, the measurements are largely confined to the region beyond $x/d = 10$. In this region the flow is fully developed, at least insofar as mean values are concerned, and so radial profiles of mean velocity and mean concentration when suitably normalized are affine. The flow is not, however, fully developed in the detailed structure of the turbulence but, since the present work is concerned only with mean values, this need be of no great concern here.

Axial distributions of the mean values for total head, velocity, and salt concentrations are presented in Figure 3 as functions of the distance downstream of the orifice. The ordinate and abscissa values are normalized using jet entrance conditions. Velocities are computed from the mean total head measurements according to the relationship

$$\bar{q}_{\max}/\bar{q}_o = \bar{U}^2_{\max}/\bar{U}^2_o \qquad (1)$$

This equation is not quite accurate for turbulent flows, but the corrections are normally small (except at the edge of the jet but here the measurements are inaccurate anyhow) and are disregarded.

The potential velocity core of the jet is seen to disappear at about 3.5 diam downstream and even earlier, at about 2 diam., the uniform concentration core disappears. This early disappearance of the uniform concentration core is not owing to molecular diffusion, but is in fact owing to intermittent turbulence in the potential velocity core. While the intermittancy is not sensed by the total head tube, its presence can readily be detected by suitable measuring devices. It can be detected, for example, with the conductivity cell used in this work.

As expected, the concentration of salt along the axis of the jet falls more rapidly than does velocity and so is indicative of the higher transport rates for material. This is confirmed in Figure 4 where the spread of material and momentum are shown in terms of the half radii of the jet. The half radius, r_3, is by definition the point in the jet where the concentration is equal to one-half the value on the axis in the same plane: r_2 is the equivalent point for the momentum, and r_o is the radius of the orifice. Beyond about 6 to 8 diam. the rate of spread for both is seen to be linear. Corrsin and Uberoi's data (*1*) for a turbulent jet of air 15°C. above ambient are shown for comparison. The Reynolds numbers of their jets were 30,000 to 50,000, or about the same as for the present jet. The rate of spread of momentum is seen to be very nearly the same for the two systems.

Radial profiles for total head and salt concentration were obtained for various distances downstream of the orifice. Except for the region nearest the orifice, that is to say, for $x/d < 10$, the mean flow of momentum and material essentially achieved the fully developed stage described earlier. The measurements will be presented in a later section of this paper.

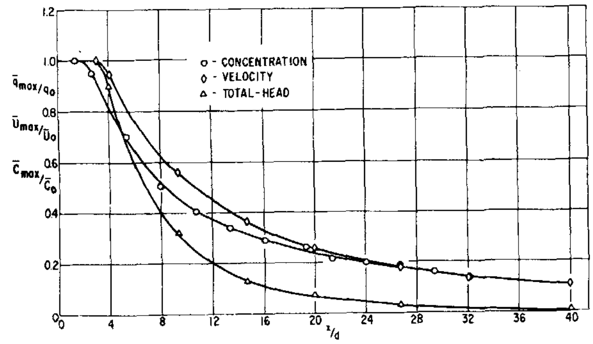

Fig. 3. Axial distribution of total head, velocity, and concentration.

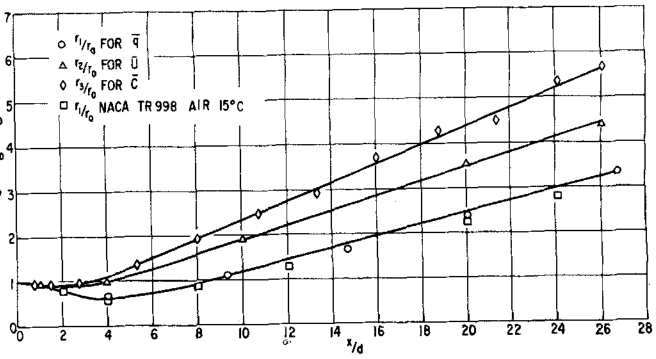

Fig. 4. Half widths of mean velocity and concentration profiles.

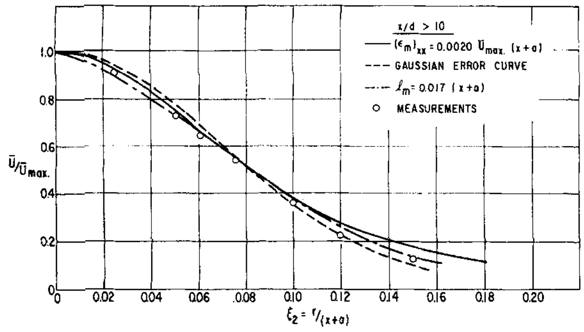

Fig. 5. Radial distribution of mean velocity.

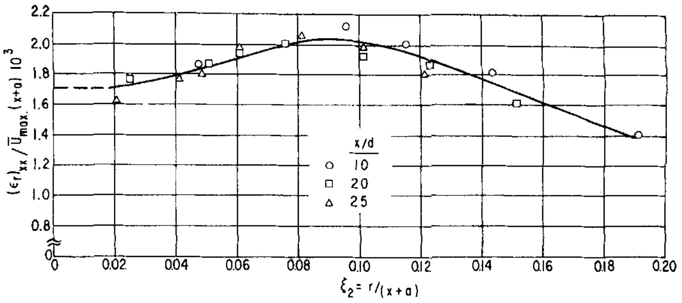

Fig. 7. Radial variation of the eddy viscosity coefficient.

PHENOMENOLOGICAL CONSIDERATIONS

Axial Distributions

For the case of a jet of fluid issuing with uniform velocity, $\overline{U}_o$, into a medium of zero velocity, it can be shown from momentum-conservation arguments and similarity assumptions that in the region where the flow is fully developed, the spread of the jet is linear with distance from the orifice $(x + a)$ and that its axial velocity decreases hyperbolically in the same direction (*13*). Figure 4 shows that the linear spread is obtained in the present work not only for the momentum but also for the concentration. The spread of the former is given in terms of the half-velocity radius, r_2, by

$$r_2/r_o = 0.163\ (x + a)/d \qquad (2)$$

The concentration, given by

$$r_3/r_o = 0.208\ (x + a)/d \qquad (3)$$

is seen to spread more rapidly than the momentum. Within the accuracy of the measurements both curves extrapolate to the same apparent origin at $x = a = 1.15d$. For a jet of air at 15°C., Corrsin and Uberoi's data (*1*) give a value of 0.166 for the constant in Equation (2). Hinze and van der Hegge Zijnen (*2*) obtained the same value for their air jets, while for the submerged jet of water Forstall and Gaylord (*9*) obtained the larger value of 0.182 which appears to be in error.

Hinze and van der Hegge Zijnen (*2*) also measured the rate of spread of material by introducing into their air jets a small amount (1%) of town gas and the rate of spread of heat by elevating the temperature of the jet relative to the ambient fluid. They obtained a value of 0.192 for the constant in Equation (3) in both cases. Forstall and Gaylord (*9*) found the constant to be 0.202 for sodium chloride in water, and from Corrsin and Uberoi's measurements (*1*) a value of 0.198 for heat in a jet of air 15°C. above ambient is obtained. The agreement between various investigators is seen to be not nearly as good as in the case of the velocity measurements. This result may reflect the sensitivity of the different measuring instruments. Hinze obtained his measurements by withdrawing samples and probing with a thermocouple, while Corrsin used a hot-wire anemometer as a resistance thermometer. Forstall used a conductivity cell. Partially at least it may also be attributable to the strong diffusive nature of the systems studied (excepting of course the liquid jet). This kind of error in measuring turbulent transport rates was pointed out earlier in the paper. Because the transport of contaminant is sensitive to the detailed structure of the turbulence, and because this structure is not likely to be the same in every system in the region studied, (compare the mesurements of Rosensweig, et al. (*14*) with those of Corrsin and Uberoi (*15*), preliminary intensity measurements for the present system check more nearly those of reference (15) the spread is greater where the turbulence is the more intense. The character of this turbulence depends somewhat on the entrance conditions and on the state of motion in the secondary fluid surrounding the jet (that is, inordinately large motion due to the proximity of walls can increase the rate of spread). Since the axisymmetric jet has not been examined in the self-preserving region in any of the work cited here, this may be particularly important.

The predicted hyperbolic decrease in axial velocity and axial concentration with increasing distance downstream is shown in Figure 3. Algebraically this decrease is given by Equations (4) and (5) respectively:

$$\overline{U}_{\max}/\overline{U}_o = 6.1\,\xi_1^{-1} \qquad (4)$$

$$\overline{C}_{\max}/\overline{C}_o = 5.0\,\xi_1^{-1} \qquad (5)$$

for the region beyond $x/d = 10$. By definition

$$\xi_1 \equiv (x + a)/d$$

The concentration data are in very good agreement with that of Weddell as reported by Hottel (*16*). Since the spread of the jet radially and the hyperbolic decrease along the axis are not independent behavior, this need not be dwelt upon.

Radial Distributions

In general, none of the various phenomenological theories of turbulent transport adequately fit experimentally determined $\overline{C}$- and $\overline{U}$-distributions over the whole jet. Some fit well near the axis while others fit better near the edges, and where one fits for momentum transport another fits better

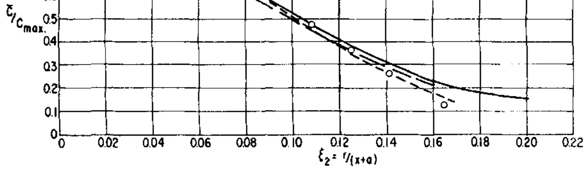

Fig. 6. Radial distribution of mean concentration.

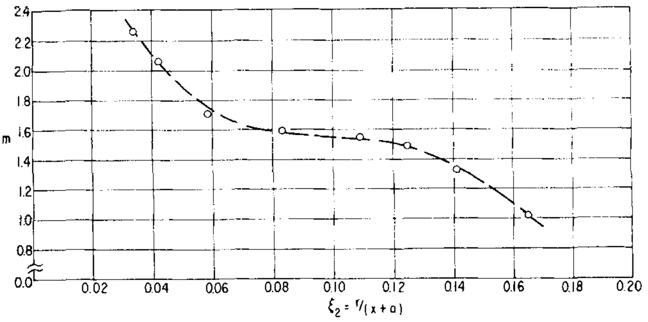

Fig. 8. Radial variation of the eddy diffusion coefficient.

Table 1. Experimentally Determined Schmidt (or Prandtl) Numbers taken from the literature

Fluid	Tracer	$(N_{Sc})_{\text{turb}}$ or	$(N_{Pr})_{\text{turb}}$	Ref.
Air	Temp.		0.74	*2*
Air	Town gas (mostly methane)	0.74		*2*
Air	Temp.		0.72	*3*
Air	Nitrogen	0.72		*5*
Air	Temp.		0.71-0.73	*1*
Water	NaCl	0.80 (avg.)		*9*
Air	Temp.		0.77	*17*
Air	Helium	0.70		*18*
Water	NaCl	0.67		Present Work

for material or heat transport. The problem is known to lie in the assumption of a gradient type of diffusion mechanism as the sole mechanism of transport across the jet. (A corollary of this is that the flow is completely turbulent but of varying intensity across the entire width of the jet.) In reality, continuously turbulent flow exists at all times only in the core of the jet (*7, 8*), a region which extends outward only to somewhere near the half-velocity radius, and turbulent transport is by a combination of two distinct mechanisms: gradient-type in the core, and convective near the edges. This then accounts for the reasonably good fit of the classical theories near the axis of the jet. It also provides an explanation for the experimentally observed fact that contaminant is more rapidly diffused than is momentum.

While the true picture of the transport mechanisms in turbulent jets (and certain other types of free turbulent flows) has been elucidated and while it contributes greatly to the understanding of turbulent shear flows in general, the state of development is such that the complexity of the calculations and measurements presently required more than offset the advantages to be gained in using it. For most practical purposes, the generally reasonable results obtained with the various phenomenological theories suffices. It is therefore worthwhile to attempt the fit of these data with several of these theories.

From the half-concentration radii data, and using the methods of Tollmien summarized in reference *13*, a value for the mixing length l_c can be calculated for the jet. The result in terms of the half-radius, r_3, is

$$l_c = 0.22 r_3 \tag{6}$$

or, in terms of the half-velocity radius

$$l_c = 0.26 r_2 \tag{7}$$

This compares with a momentum mixing length of

$$l_m = 0.21 r_2 \tag{8}$$

calculated in a similar manner from the half-velocity radii data.

From Equations (6) and (8), the radial distribution for the mean velocities and the mean concentrations respectively were calculated. The results are presented in Figures 5 and 6. It is seen there that such an operation leads to values of $\overline{U}/\overline{U}_{\max}$ and $\overline{C}/\overline{C}_{\max}$ which, in general, are less than those obtained experimentally in the central region of the jet and greater than those obtained experimentally near the edge. This is usually the case with the mixing length theory. Nevertheless, the fit of the theory to the velocity measurements is quite good over much of the jet. The concentration measurements on the other hand are fitted poorly everywhere.

As is evident from Figure 6, a somewhat better fit of the experimental $\overline{C}$-distribution is obtained by a constant eddy diffusion coefficient assumption fitted to the half-concentration radii data. From these data a value of

$$\epsilon_c = 0.0030\, \overline{U}_{\max} (x + a) \tag{9}$$

was calculated.

A similar assumption for the $\overline{U}$-distribution leads to

$$\epsilon_m = 0.0020\, (x + a) \tag{10}$$

but as seen in Figure 5, the fit is, if anything, worse than that obtained with the mixing length theory, especially near the edges of the jet.

The best fit of the experimental $\overline{C}$-data is seen in Figure 6 to be given by a Gaussian error curve with

$$\overline{C}/\overline{C}_{\max} = \exp\left[-\,0.492\, (r/r_2)^2\right] \tag{11}$$

The error curve for the experimental $\overline{U}$-data is

$$\overline{U}/\overline{U}_{\max} = \exp\left[-\,0.694\, (r/r_2)^2\right] \tag{12}$$

but this does not give nearly as good a fit as does the mixing length theory either.

Turbulent Schmidt Number

The ratio ϵ_m/ϵ_c is the turbulent Schmidt number which for the present system, with the half-radii data is equal to 0.67. This value is somewhat lower than obtained by others as may be seen from Table 1.

However, when the eddy diffusivities are calculated from the radial data, it is seen that no single value for either of these quantities can be made to fit the whole distribution. Neither is constant across the jet. These results are shown in Figures 7 and 8, respectively. A value for $(N_{Sc})_{\text{turb}} > 0.67$ will give a better fit to the data in Figures 5 and 6 in the core of the jet and so will be more in agreement with the data in Table 1. There will be a corresponding deterioration in the fit at the edges however.

CONCLUSIONS

The results of this work are in good agreement with those of others: material diffuses turbulently neither more rapidly nor more slowly in liquid systems (relative to momentum) than it does in gaseous systems. Small differences are noted but these can be attributed either to differences in measuring techniques or to differences in the detailed structure of the turbulence which is known to vary from system to system. Preliminary concentration fluctuation measurements (not reported here) when compared with equivalent data in the literature shows variations also.

The mean value measurements were best fitted with a Gaussian error curve in the case of the concentration field and the mixing length theory in the case of the velocity field. Usually best results are obtained with a constant eddy diffusivity in the latter case. A value of 0.67 has been obtained for the turbulent Schmidt number in the usual manner, that is, from the half-radii data, but it is shown that this varies appreciably across the jet.

Finally, because the mean concentration measurements were determined by taking average values of instantaneous values (which actually were measured) and these are consistent with the literature, the measuring equipment appears suitable for making measurements of the fluctuating concentration. Of course mere agreement of mean values is not a sufficient test of the adequacy of the instrument. The frequency response of the cell and, more particularly, the volume element of the probe sees has to be determined precisely. The latter point is important for conductance cells have been much criticized for seeing volume elements which are not small compared to the dissipative scale of the turbulence.

NOTATION

a = apparent origin of jet measured from orifice

$\overline{C}$ = mean concentration at a point (relative to ambient fluid)

d = orifice diameter

l = transport mixing length

N = dimensionless number

$\overline{q}$ = mean dynamic pressure at a point

r = radial distance from jet axis

r_o = $(\equiv 1/2d)$

r_1 = value of r for which $\overline{q} = 1/2\ \overline{q}_{\max}$

r_2 = value of r for which $\overline{U} = 1/2\ \overline{U}_{\max}$

r_3 = value of r for which $\overline{C} = 1/2\ \overline{C}_{\max}$

$\overline{U}$ = axial component of mean velocity at a point

x = axial distance from orifice

ξ_1 = $(x + a)/d$

ξ_2 = $r/(x + a)$

ϵ = turbulent diffusion coefficient

Subscripts

c = material

m = momentum

max = maximum value at a cross-section

o = at the orifice

Pr = Prandtl number

Re = Reynolds number

Sc = Schmidt number

turb = turbulent

LITERATURE CITED

1. Corrsin, S., and M. S. Uberoi, *Natl. Advisory Comm. Aeronaut. Tech. Rept. 998* (1950).
2. Hinze, J. O., and B. G. van der Hegge Zijnen, *Appl. Sci. Research,* **1A**, 435 (1949).
3. Ruden, P., *Naturwissenschaften,* **21**, 375 (1933).
4. Reichardt, H., *Forsch. Gebiete Ingenieurw.*, 414 (1951).
5. Keagy, W. R., and A. E. Weller, *Proc. Heat Trans. Fluid Mech. Inst.*, p. 89, Berkeley, California (1949).
6. Forstall, W., Jr., and A. H. Shapiro, *J. Appl. Mech.*, **17**, 399 (1950).
7. Corrsin, S., and A. L. Kistler, *Nat'l. Advisory Comm. Aeronaut. Tech. Note 3133* (1954).
8. Townsend, A. A., "The Structure of Turbulent Flow," Cambridge University Press, New York (1956).
9. Forstall, W., Jr., and E. W. Gaylord, *J. Appl. Mech.*, **22**, 161 (1955).
10. Prausnitz, J. M., and R. H. Wilhelm, *Rev. Sci. Instr.*, **27**, 941 (1956).
11. Cairns, E. J., Ph.D. dissertation, Univ. of Calif., Berkeley, California (1959).
12. Manning, F. S., Ph.D. dissertation, Princeton Univ., Princeton, New Jersey (1959).
13. Hinze, J. O., "Turbulence," McGraw-Hill, New York (1959).
14. Rosensweig, R. E., H. C. Hottel, and G. C. Williams, *Chem. Eng. Sci.*, **15**, 111 (1961).
15. Corrsin, S., and M. S. Uberoi, *Nat'l. Advisory Comm. Aeronaut. Tech. Rept. 1040* (1951).
16. Hottel, H. C., "Fourth Symposium on Combustion," pp. 97-113, Williams and Wilkins, Baltimore, Maryland (1953).
17. Cleeves, V., and L. M. K. Boelter, *Chem. Eng. Progr.*, **43**, 123 (1947).
18. Forstall, W., Jr., Sc.D. dissertation, Mass. Inst. of Technol., Boston, Massachusetts (1949).

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The Prediction of Vapor-Liquid Equilibria Using a Theory of Liquid Mixtures

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Much investigation in the field of thermodynamics of liquid solutions has been undertaken in connection with vapor-liquid equilibrium phenomena. The nature of such investigations has been determined by the objective. One objective is to determine whether the experimental data, specifically the measured relationship between liquid and vapor compositions, temperature in an isobaric system or pressure in an isothermal system, and vapor pressures of the pure substances, are thermodynamically consistent. The objective is accomplished by determining whether or not the variables follow the Gibbs-Duhem relationship, and the degree to which they do is a measure of the reliability of the experimental data. Such investigations are referred to as correlations.

The other objective is to predict the vapor-liquid equilibria from a limited amount of experimental data. The more modest attempts have been aimed at predicting the most essential data, vapor composition as a function of liquid composition, from other more easily obtained experimental data. An example is the calculation of these data from experimental determinations of vapor pressures of pure components, and the boiling temperatures of mixtures at constant pressure.

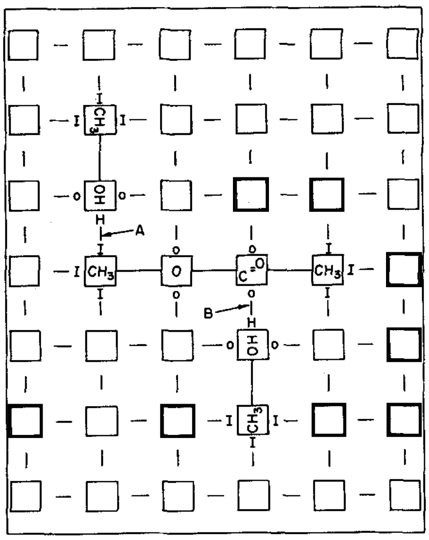

Fig. 1. Interaction between molecules on a lattice.

The more ambitious attempts at prediction involve calculation of complete equilibria from the properties of the pure substances and a measure, independent of experimental vapor-liquid equilibrium determinations, of their behavior in mixtures. The simple example of such a method is a common Raoult's-law prediction in which the vapor pressures are used and it is assumed that the behavior upon mixing is ideal. Where this assumption can-